

# Tools for RHIC: Review of Models\*

K. Werner

Laboratoire de Physique Subatomique et des Technologies Associées (SUBATECH)  
Université de Nantes, IN2P3/CNRS, Ecole des Mines de Nantes  
4, rue Alfred Kastler, F-44070 Nantes Cedex 03, France.

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## Abstract

We discuss the present status of microscopic models for RHIC, with an emphasis on models being realized via the Monte Carlo technique. This review is to a large extent based on the OSCAR3 workshop, where general concepts and new trends in this field have been discussed.

## 1 Introduction

Although very interesting data have been collected during the SPS program of heavy ion physics, no clear quantitative conclusions can be drawn concerning the formation (or not) of a quark gluon plasma. A well established theory exists (QCD), however, technical difficulties prevent a direct application of the theory to understand data. Effective theories have been proposed to overcome these difficulties, as well as very simple qualitative models, which might as best be called “theory inspired”. On the other hand, the so-called “event generators” or “Monte-Carlo codes” have been introduced, which by definition provide randomly generated “events”, characterized by a certain number of particles of different types with given momenta. One aim of this paper is to discuss general strategies how such event generators should be constructed in order to provide useful tools to understand experimental data.

There is some effort to be done, but it is worth it, because finally MC codes are absolutely necessary to understand data. As an illustration, we plot in fig. 1 yields of different hadrons as a function of the hadron mass for different reactions: electron-positron annihilation at 91 GeV, pp scattering at 17 GeV, and AuAu scattering at 200 GeV. The curves are arbitrarily normalized. We observe a very interesting and unexpected result: all the spectra are roughly exponential, and the spectra for heavy ion collisions agree with the result for

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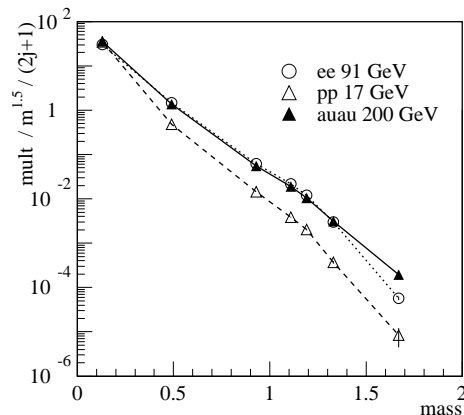


Figure 1: Hadron yields as a function of the hadron mass for different reactions: electron-positron annihilation at 91 GeV (open dots), pp scattering at 17 GeV (open triangles), and AuAu scattering at 200 GeV (full triangles).

electron-positron. Here, all the results are based on calculations, and so we know that the dynamics of electron-positron annihilation is completely different to the one in heavy ion collisions. So one cannot draw any conclusion concerning a plasma formation based on the fact that strange particles and in particular (multi-) strange baryon production is enhanced, since the same effect may be due to a completely different mechanism (as in electron-positron annihilation).

## 2 OSCAR Philosophy

Monte-Carlo simulations are often criticized of not being well documented, having no clear physical basis, being constantly modified, being not publicly available, and so on, which makes all evaluation of the quality of such an approach impossible. In order to improve the situation, OSCAR was founded a couple of years ago, which is first of all a series of workshops, hold every one or two years, as well as working groups and a permanently updated WEB page<sup>1</sup>. In the working groups, mainly technical issues have been discussed, like standards for input/output, test for individual moduls, and so on. The first two workshops were held in Brookhaven in 97 and 99, organized by Y. Pang and M. Gyulassy, the third one took place in Nantes in 2000, organized by Y. Schutz and K. Werner.

At the first two workshop the general philosophy was still based on the fact that MC models can hardly be defined by equations, they were mostly defined in a algorithmic way, and therefore the only way to control these codes is good documentation, definition of standards, accessibility, modular structure, all this

<sup>1</sup> <http://nt3.phys.columbia.edu/OSCAR>

in particular to allow intensive testing of these codes.

Starting with the third workshop a new philosophy has been discussed, in an attempt to make the event generators much more transparent, and thus to provide really useful tools for analyzing data. So the model should be simply defined by equations, and the computer code should be just the technical means to solve these equations, briefly:

$$\begin{aligned}\text{model} &= \text{equations,} \\ \text{MC code} &= \text{solution of the equations.}\end{aligned}$$

It is clear that these equations building the basis of the MC code will (probably) never be directly derived from first principles, this is also not necessary. One may construct some effective theory which may be simply inspired by the “true” theory, but the next step, the MC implementation, has to be rigorous. The advantage is obvious: it is difficult to have any meaningful discussion about some code where the physical basis is not well defined. When the code however is nothing but the numerical treatment of some very well defined (even if not fundamental) effective theory, one is at least able to have some physics discussion, develop further promising developments, and eliminate those ones based on wrong physical ideas. There was very little progress in this direction during the past years.

### 3 The Different Stages of Heavy Ions Collisions

Unfortunately there does not exist a single formalism being able to account for a complete nucleus-nucleus collision. Rather we have to – at least for the moment – to divide the reaction into different stages (see fig. 2) and try to understand

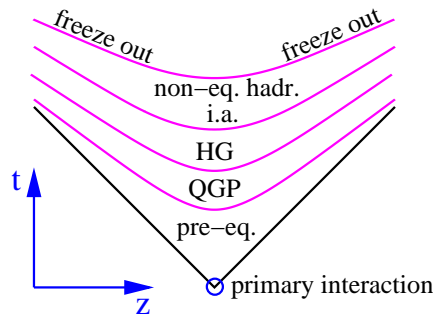


Figure 2: The different stages of heavy ion collisions.

the different stages as good as possible.

There is first of all the primary interaction when the two nuclei pass through each other. Since at very high energies the longitudinal size is due the gamma

factor almost zero (of the order 0.1 fm at RHIC), all the nucleons of the projectile interact with all the nucleons of the target instantaneously. In such a primary interaction many partons are created, which interact (in the pre-equilibrium stage) before reaching an equilibrium, referred to as quark-gluon plasma. The system then expands, passing via phase transition (or sudden crossover) into the hadron gas stage. The density decreases further till the collision rate is no longer large enough to maintain chemical equilibrium, but there are still hadronic interactions till finally the particles “freeze out”, i.e. they continue their way without further interactions. Different theoretical approaches have been

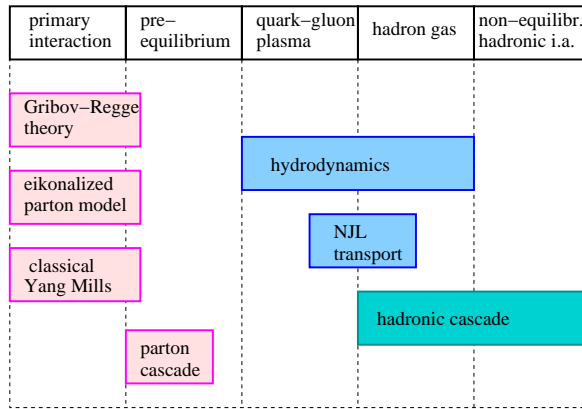


Figure 3: The different stages of heavy ion collisions and the corresponding range of validity of theoretical approaches.

proposed, which are valid only for certain stages of the collision, as indicated in fig. 3. We will present the details in the following sections. This discussion will certainly be quite incomplete, with few exceptions we restrict ourselves to topics having been discussed at OSCAR 3.

## 4 The primary interaction

### 4.1 Longitudinal Excitation

Let us start the discussion with an approach which is widely used today to simulate the primary interactions, but which is definitely wrong: the longitudinal excitation. At low energy nucleon-nucleon scattering, a typical reaction is the excitation of nucleon resonances via the exchange of a meson. So one might be tempted to generalize this mechanism to high energies: two nucleons interact via the exchange of “something” which causes the two nucleons to be excited to strings, the latter ones considered to be the high energy generalization of excited nucleon states. However, such an extrapolation from low to high energies is simply wrong, due to kinematical reasons. Let us consider two-body

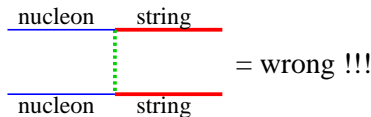


Figure 4: Longitudinal excitation (which is kinematically impossible).

kinematics: two incoming particles with four-momenta  $p_1$  and  $p_2$  interact in a non-specified way and produce two outgoing particles with momenta  $p_3$  and  $p_4$ . The transferred momentum is defined to be  $q = p_1 - p_3$ . A short calculation shows:

$$q^2 = q_{\perp}^2 + O\left(\frac{1}{s}\right),$$

where  $q_{\perp}$  is the transverse component of the transferred momentum, orthogonal to  $p_1$  and  $p_2$ . At high energies ( $s \gg 1 \text{ GeV}^2$ ), the transferred momentum is consequently purely transverse, there is no transfer of longitudinal momentum. String excitation, on the other hand, requires a transfer of longitudinal momentum in order to allow for string with non-zero mass.

All this has been known since forty years, which does not prevent people from coming up with models based on this wrong idea, or using such models in trying to understand data.

## 4.2 Yang-Mills Equations

For very large nuclei and correspondingly high parton densities screening will be most effective, and therefore soft physics can be completely accounted for by assuming random color sources moving along the light cones, the latter ones generating chromoelectric fields calculable by solving the corresponding classical Yang-Mills equations [1].

This is an interesting theoretical idea, although it is not clear how to construct an “event generator” based on such an approach, so we do not want to discuss any details here.

## 4.3 The Parton Model

The parton model approach to nucleon-nucleon scattering amounts to presenting the partons of projectile and target by momentum distribution functions,  $f_i$  and  $f_j$ , and calculating inclusive cross sections for the production of parton jets as a convolution of these distribution functions with the elementary parton-parton cross section  $d\hat{\sigma}_{ij}/dp_{\perp}^2$ , where  $i, j$  represent parton flavors.

This simple factorization formula is the result of cancelations of complicated diagrams (AGK cancelations) and hides therefore the complicated multiple scattering structure of the reaction, which is finally recovered via eikonalization procedure. The latter one makes the approach formally equivalent to the Gribov-Regge one, to be discussed later. Generating events and particle production is

not at all evident in this approach. The Pythia-method [2] amounts to generating the first elementary interaction according to the inclusive differential cross section, then taking the remaining energy for the second one and so on. In this way, the event generation will reproduce the theoretical inclusive spectrum for hadron-hadron interaction (by construction).

Concerning nucleus-nucleus collisions, one usually assumes the proton-proton cross section for each individual nucleon-nucleon pair of a  $AB$  system. Nuclear screening effects may be taken into account by using  $A$ -dependent parton distribution functions,  $f_i^A$  and  $f_j^A$ , rather than the ones used for nucleon-nucleon scattering (this is usually referred to as shadowing).

The HIJING model [3] is constructed along these lines, with the additional feature of considering the energy loss of partons due to final state interactions.

#### 4.4 Gribov-Regge Theory

Gribov-Regge theory (GRT) has been developed well before QCD, but it is even today more relevant than ever, with HERA giving the opportunity to test and verify the different aspects of this approach. There is no strict derivation from first principles, one can just follow some “QCD inspired arguments” to write down an expression for the elastic scattering amplitude for nucleon-nucleon scattering in terms of many elementary scatterings, which easily generalizes to nucleus-nucleus scattering. From there on, one uses strictly the rules of quantum mechanics to obtain a multiple scattering approach for particle production. The key ingredient is the fact that the cross section is obtained from squaring the amplitude, so one obtains partial contributions as shown in fig.5. In the

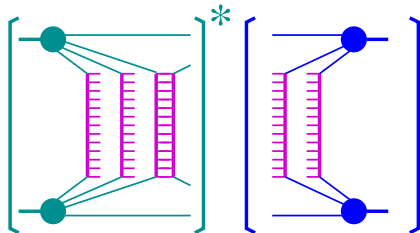


Figure 5: An interference term in GRT.

example shown in the figure, we have a diagram with two inelastic interactions (symbolically shown as “comb”) interfering with the diagram with two inelastic and one elastic scattering (symbolically shown as “ladder”). So we have classes of interfering contributions, which have to be summed up. Ignoring energy conservation, one obtains a simple formula for the inelastic cross section:

$$\sigma_{\text{inel}}(s) = \int d^2b \sum_m \frac{f(s, b)^m}{m!} e^{-f(s, b)}. \quad (1)$$

The function  $f$  is related to the inclusive cross section as

$$f(s, b) = \sigma_{\text{incl}}(s) A(b), \quad (2)$$

with some function  $A$  representing the impact parameter dependence. The equations (1, 2) provide the link between the parton model and GRT: since the parton model only provides inclusive cross sections, one uses these formulas to introduce multiple scattering. Or one may say it the other way round: GRT models introduce hard scattering via inclusive cross sections calculated in the parton model.

Models based on the Gribov-Regge approach are QGS [4], DPM [5, 6], VENUS [7].

#### 4.5 String Fusion

Colliding heavy nuclei at very high energies will produce a large number of strings, which can certainly not be considered completely independent as in standard GRT. A phenomenological approach consists of considering string fusion, whenever pairs of strings are coming close to each other [8]. Fused strings are assumed to behave like ordinary strings simply with a increased string tension. This will, for example, lead to an enhanced production of multi-strange particles.

#### 4.6 Nuclear Shadowing

There exists also a formally well defined treatment of high density effects via the so-called enhanced diagrams, which amounts to a generalization of the basic Gribov-Regge theory by taking into account Pomeron-Pomeron interactions. This is responsible not only for nuclear shadowing, but also for diffractive scattering in  $pp$  but also in deep inelastic scattering. In fact it is very important to have a consistent picture of  $pp$  and DIS, in general and in particular for diffractive scattering [9]. The latter one reveals a reduction of the Pomeron flux compared to a naive expectation from GRT, which then leads to a reduced screening. To say it differently, one has to go beyond simple lowest order enhanced diagrams (triple Pomerons).

#### 4.7 Proper Energy Sharing in GRT

Although GRT is a useful starting point to describe high energy hadronic and nuclear scattering, there are serious drawbacks. As pointed out in [10], GRT is lacking a consistent picture for the calculation of the cross section formulas and for particle production. The problem is the energy sharing between the individual scatterings. Doing this properly makes the approach considerably more complicated, and therefore the standard approach is to ignore energy sharing at this level, and considering it later when it comes to particle production. This is clearly not consistent, and in fact the error due to neglecting the energy sharing is quite large: the width for the distribution of the number of multiple

scatterings is roughly doubled when energy conservation is ignored. This can be compensated by making a second mistake which amounts to ignoring the increase of multiplicity fluctuations due to cutting enhanced diagrams properly. A new model NEXUS has been proposed recently[10], where energy conservation is treated properly on all levels. In addition, lowest order enhanced diagrams are considered. Soft and hard scattering is treated such that there is a smooth transition between the two regimes, and in particular a dependence on some transverse momentum cutoff  $p_0$  can be avoided.

## 5 Pre-Equilibrium

The partons created in the primary interactions are certainly far from equilibrium, and it is desirable to understand microscopically the equilibration of the system, in other words the formation of a quark gluon plasma. This is a difficult task, since for example at RHIC energies there is still a large soft component. Nevertheless it is useful to study the evolution of partonic systems based on pQCD, ignoring soft physics.

### 5.1 Parton cascade

A parton cascade amounts to considering partons as classical particles which move on straight line trajectories, where binary interactions are defined via parton-parton cross sections calculated in the framework of perturbative QCD [11].

One has to carefully regard the range of validity of this approach: it is not meant to treat the primary interactions, where quantum mechanical interference should play a crucial role, so one may start the calculation once a system of incoherent classical partons have been established. On the other end, one should not stretch the perturbative treatment too far: perturbative calculations require large momentum transfer which is not any more guaranteed if the interaction energy is getting too low.

A parton cascade is often referred to as the solution of a Boltzmann equation. In this case one has to work with test particles rather than real particles and one has to make sure that the number of test particles is sufficient to really provide a solution of the equation [12] (which is in accordance with the OSCAR principle “code = solution of equation”). It turns out that the number of test particles has to be much larger than the number of real particles, which prevents a cascade of real partons to be considered as a solution of a Boltzmann equation. Other than this “particle subdivision test” other tests like “box tests” should be performed to make sure that a cascade algorithm solves really a transport equation.

### 5.2 Parton equilibration

There are also analytical approaches attempting to understand parton equilibration [13]. Starting from a parton density given by the parton model with



some cutoff  $p_0$ , one calculated the evolution of the system based on a transport equation of the type

$$p\partial f = -\frac{pu}{\tau}(f - f_{\text{eq}}),$$

with  $\tau$  being the relaxation time, which is given as  $\tau = 1/\sigma n$ , with  $\sigma$  being the in-medium cross section, and  $n$  the parton density. Comparing RHIC and LHC results, the density will be of course bigger for LHC, and in both cases decreasing with time. Due to the larger screening, the cross section will be smaller for LHC compared to RHIC, and in both cases increasing with time. Taking all together one obtains the time dependence of the relaxation time which peaks around 1.5 fm, with the LHC value being somewhat bigger than the RHIC one. One finds a free streaming up to around 1.5 fm, then the equilibration starts.

### 5.3 Parton Energy Loss

Partons created in hard primary interactions will lose energy when traversing matter, the latter one being projectile or target nuclei or quark matter [14, 15]. The formulas for the energy loss obtained so far were obtained in the limit of either large or small system sizes ( $L$ ). New developments have been reported [16], which allow a calculation of the energy loss for arbitrary  $L$ , by using a systematic expansion in opacity ( $L/\lambda$ ), where  $\lambda$  is the mean free path of the parton. It turns out that the expansion converges rapidly, with the second order already being a very small correction compared to the first order one.

## 6 Equilibrium and Post-Equilibrium

We are now discussing the final stage of the collision, consisting of QGP phase, the hadron gas phase, and the very final stage where the hadrons still interact, but they do not form an equilibrated system any more. We do not treat these three stages individually, because the models to be discussed latter treat usually more than just one stage.

### 6.1 Hydrodynamics

The final aim of all the efforts in the field of ultra-relativistic heavy ion collisions is the creation of a thermalized system of quarks and gluons. Provided such an equilibrium has been established, one may use hydrodynamics, which is a macroscopic approach based on energy-momentum conservation and local thermal equilibrium. Hydrodynamical calculations have been used since a long time, either assuming particular symmetries and using analytical methods [17], or full 3-dimensional calculations numerical calculations [18]. Recently a new technique has been proposed, the so-called smoothed particle hydrodynamics [19], where fields  $\rho(x)$  are represented by particles as  $\rho_P(x) = \sum_b \nu_b \delta(x - x_b)$ ,

and then smoothed:

$$\rho(x) \rightarrow \rho_{SP}(x) = \int \rho_P(x) W(x - x') dx' = \sum_b \nu_b W(x - x_b),$$

with some smoothing kernel  $W$ . The advantage is that the hydrodynamical equations are transformed into a system of ordinary differential equations, which can be solved by applying standard methods. In this way one may perform 3-dimensional calculations much faster than with traditional methods.

## 6.2 Hadronization

There are several attempts to treat at least the region around the phase transition in a microscopic way. A possibility is to apply transport theory based on the **NJL model** [20], which is an effective theory with a point-like interaction between two quarks (gluons are not considered explicitly). The model allows also for hadron production like quark plus anti-quark goes into meson plus meson. The dynamics is crucially affected by the density and temperature dependence of quark and hadron masses, one observes for example the formation of droplets of quark matter rather than homogeneous matter of lower density, since the latter one would imply higher quark masses.

A completely different hadronization scenario has been proposed based on the **confinement** mechanism [21], again ignoring gluons. Quarks are considered to be classical particles, their dynamics being determined by a classical Hamiltonian. The latter one contains a string potential and color factors which force the quarks to form resonances, which subsequently decay into hadrons.

Another alternative approach is the hadronization via **coalescence** [22]. Again, starting from a quark-anti-quark plasma, hadronic resonances are formed based on coalescence, with a subsequent decay into hadrons.

## 6.3 Hadronic Transport Theory

Once a purely hadronic system has been established, a microscopic treatment based on binary hadronic interactions is feasible. Here, hadrons propagate on classical trajectories and interact according to hadron-hadron scattering cross sections. If possible, parameterizations of measured cross sections are used. A couple of models have been constructed along these lines, like UrQMD [23, 24], ART [25], JAM [26]. Unfortunately, not all the necessary cross sections have been measured to a sufficient precision, and correspondingly the above-mentioned approaches differ by using different model assumptions for the cross sections. We emphasize again that hadronic transport codes are a useful tool to treat the final stage of a heavy ion collision, but not for the primary interaction.

## 7 Outlook

Historically, most models have been first developed for a certain aspect of the collision, and have later been extended to include more and more features in

order to give a complete description of the collision. This is not a good way to proceed, since in this way the models have “strong parts” and “weak parts”, and the whole approach is not very reliable. A better way is to consider “moduls”, describing just one aspect of the collision in the most realistic fashion, and combine such moduls. Examples, which have been discussed at the OSCAR3 workshop, are a combination of hydrodynamics and a hadronic cascade (hydro+UrQMD [27]) or a combination of a primary interaction model and hydrodynamics (NEXUS+SPHERIO[19]).

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